A 1:2 internal resonance

▷ **A classical case with**

- Period doubling bifurcations
- Neimark-Sacker bifurcations

▷ **Applications**

- Spring pendulum, ships, surface waves, arches...

  [Nayfeh *et al.*, 1989, 2000..., Tien *et al.* 1994]

- Shells, Cymbales, Gongs, Steel Pans

  [Thomas *et al.* 2005, 2007, Achong 90']

▷ **Two quadratically coupled oscillators**

$$\begin{cases} \ddot{u}_1 + \mu_1 \dot{u}_1 + \omega_1^2 u_1 + \beta_1 u_1 u_2 = F_1 \cos \Omega t \\ \ddot{u}_2 + \mu_2 \dot{u}_2 + \omega_2^2 u_2 + \beta_2 u_1^2 = F_2 \cos \Omega t \end{cases}$$

▷ **1:2 internal resonance**

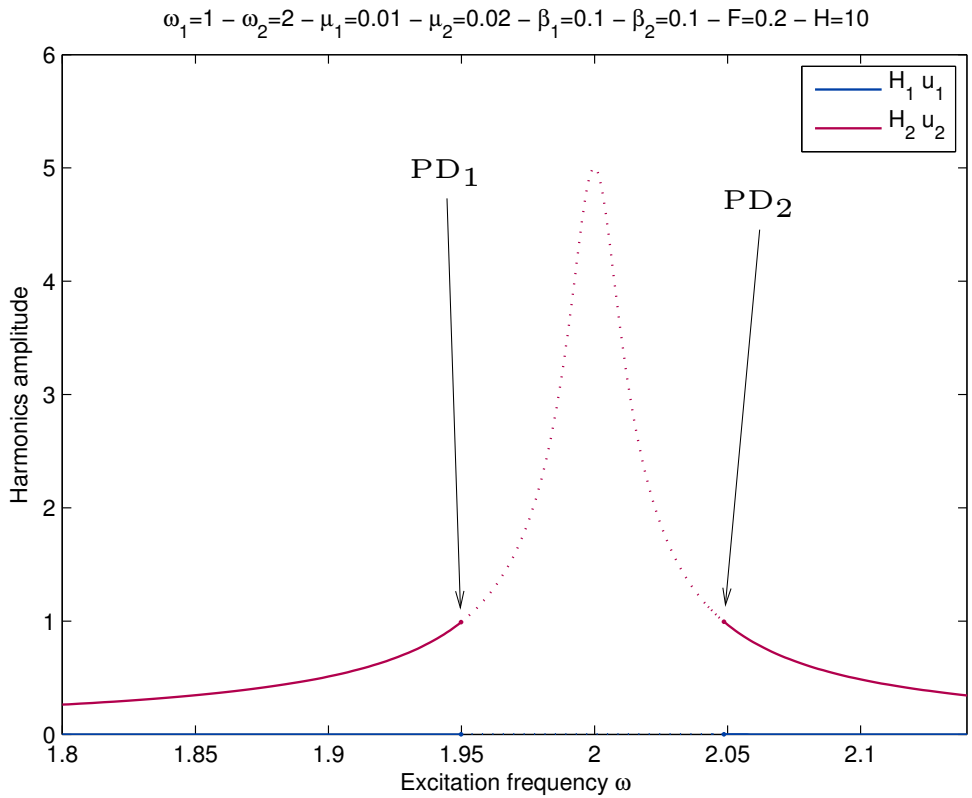
$$\omega_2 \simeq 2\omega_1$$

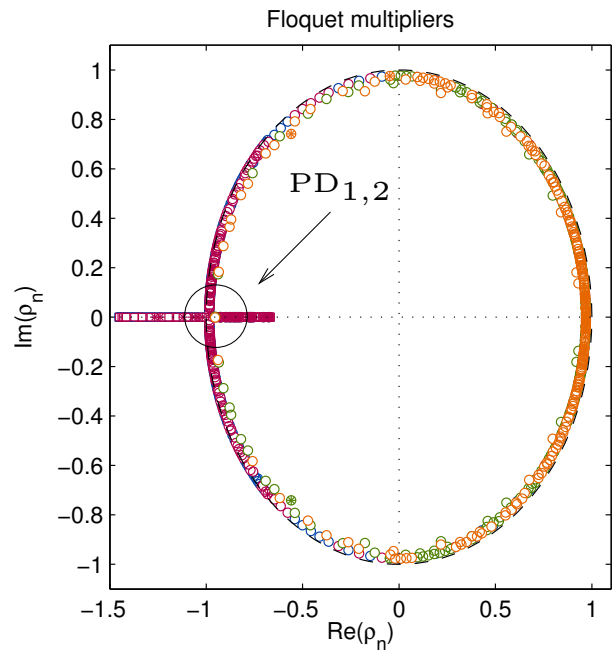
By courtesy of Vierlo Sotham

Excitation of the second mode: P.D. bifurcations

$$\begin{cases} \ddot{u}_1 + \mu_1 \dot{u}_1 + \omega_1^2 u_1 + \beta_1 u_1 u_2 = 0 \\ \ddot{u}_2 + \mu_2 \dot{u}_2 + \omega_2^2 u_2 + \beta_2 u_1^2 = F \cos \Omega t \\ \omega_2 \simeq \omega_1, \qquad \Omega \simeq \omega_2 \end{cases}$$

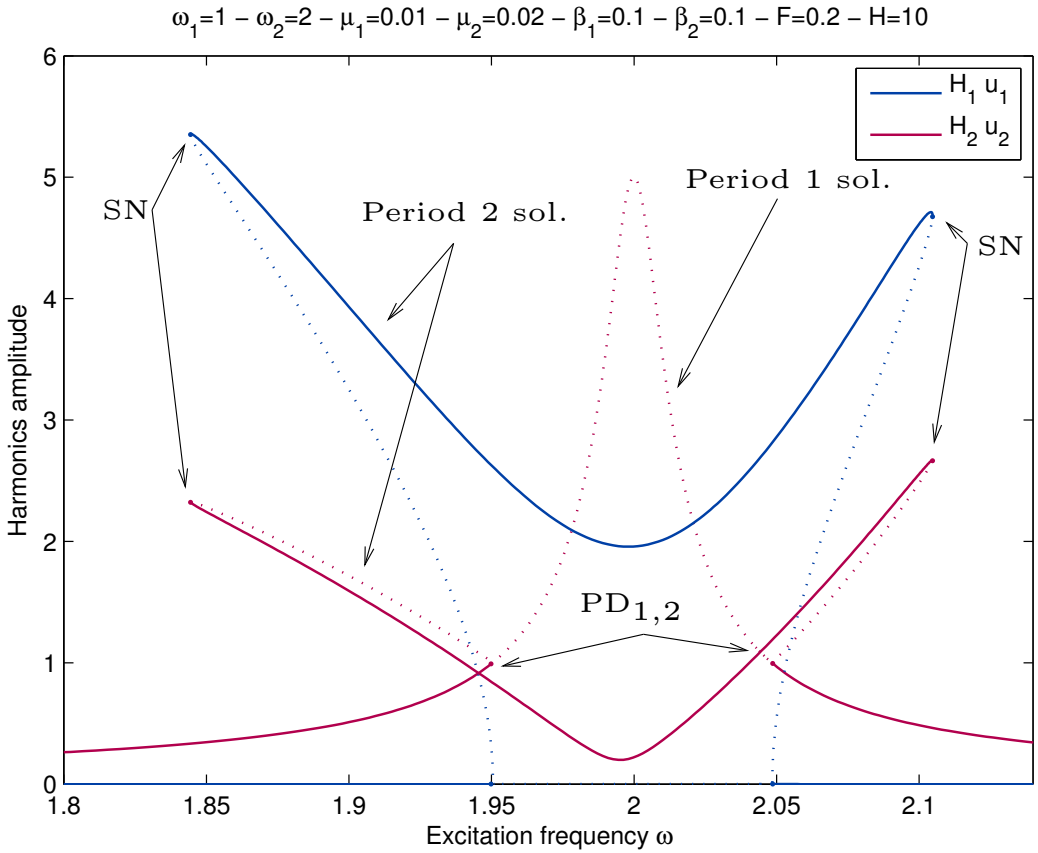
$$\begin{cases} u_1(t) \simeq 0 \\ u_2(t) \simeq a_2 \cos(\Omega t + \varphi_2) \end{cases}$$



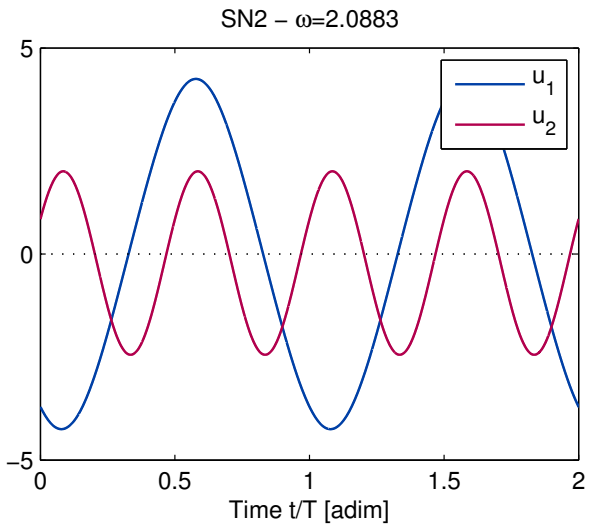


Two period-doubling bifurcations

Excitation of the second mode: resonance curve



$$\begin{cases} u_1(t) \simeq a_2 \cos\left(\dfrac{\Omega}{2}t + \varphi_1\right) \\ u_2(t) \simeq a_2 \cos(\Omega t + \varphi_2) \end{cases}$$



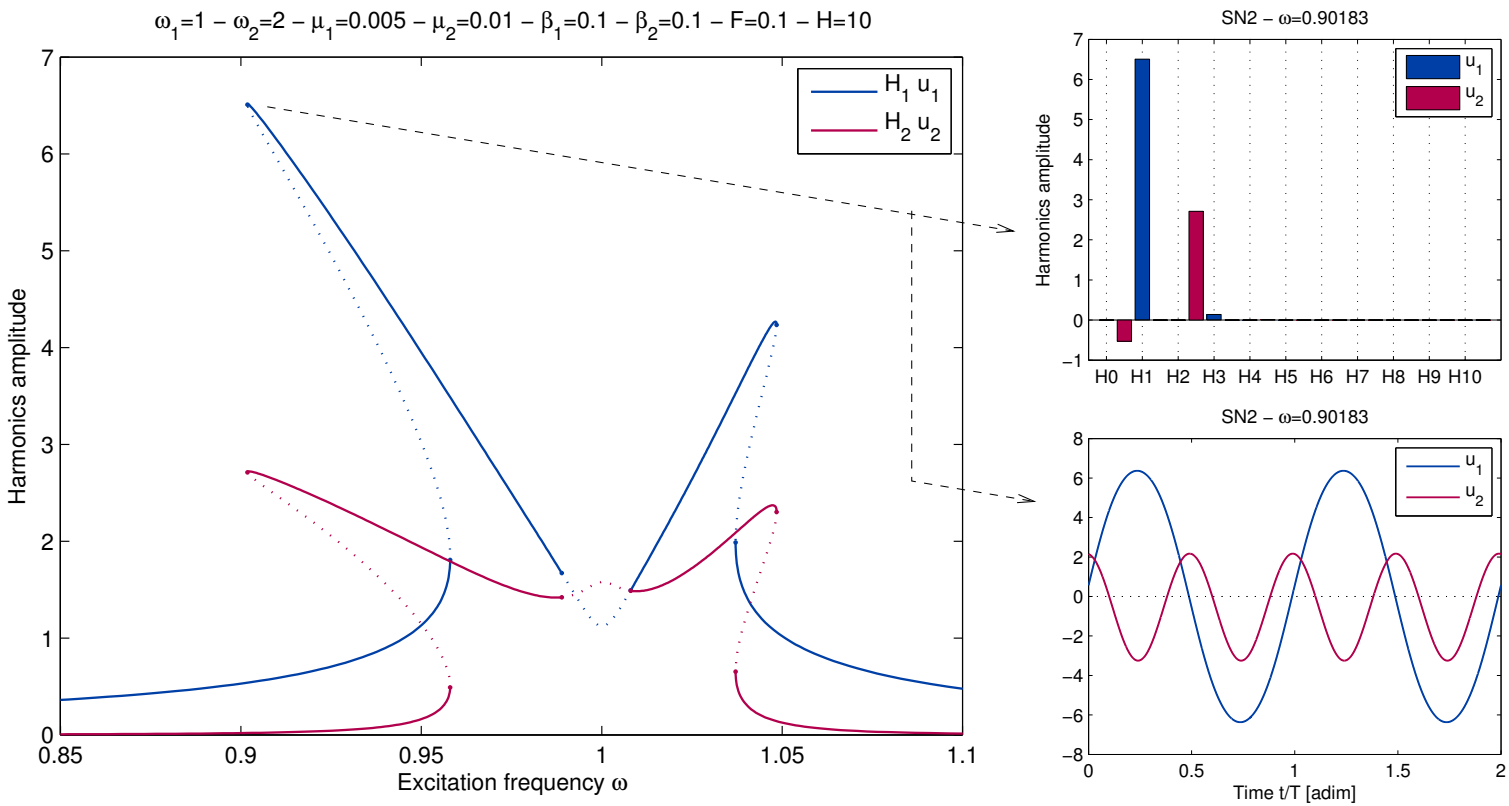
▷ **Comparisons with AUTO:** (not shown)

Branches and bifurcations are well predicted

Excitation of the first mode: resonance curve

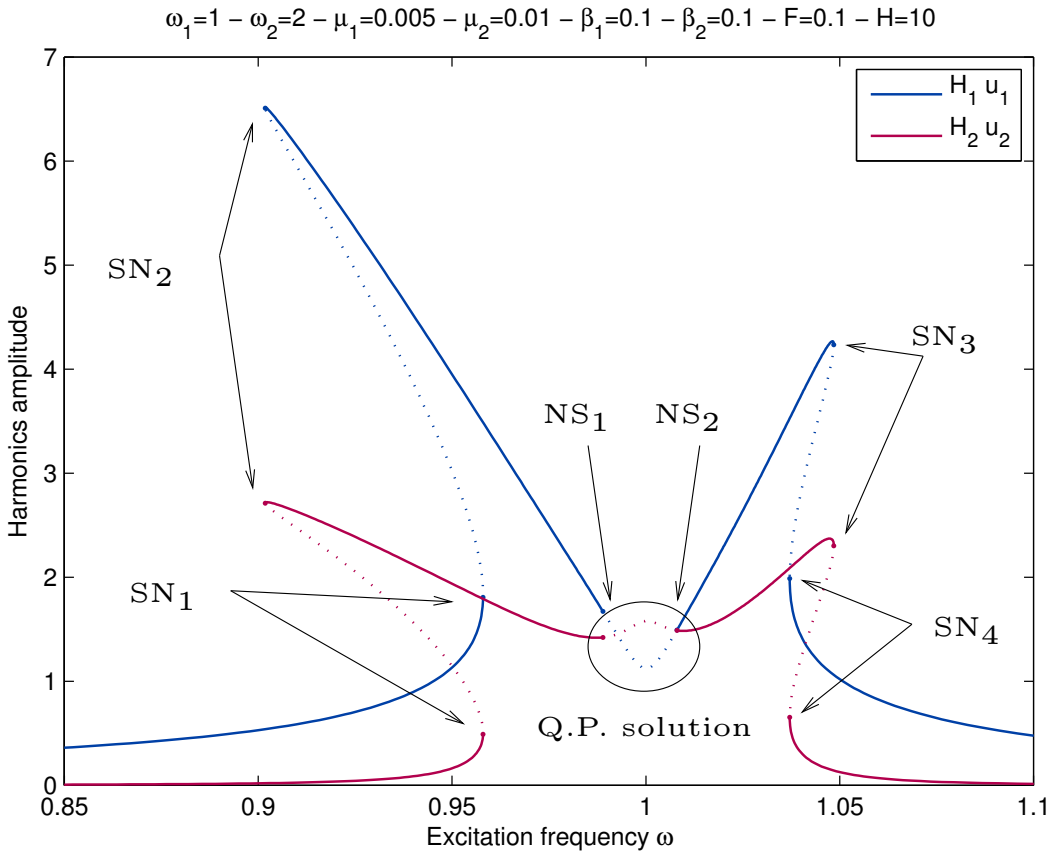
$$\begin{cases} \ddot{u}_1 + \mu_1 \dot{u}_1 + \omega_1^2 u_1 + \beta_1 u_1 u_2 = F \cos \Omega t \\ \ddot{u}_2 + \mu_2 \dot{u}_2 + \omega_2^2 u_2 + \beta_2 u_1^2 = 0 \\ \omega_2 \simeq \omega_1, \qquad \Omega \simeq \omega_1 \end{cases}$$

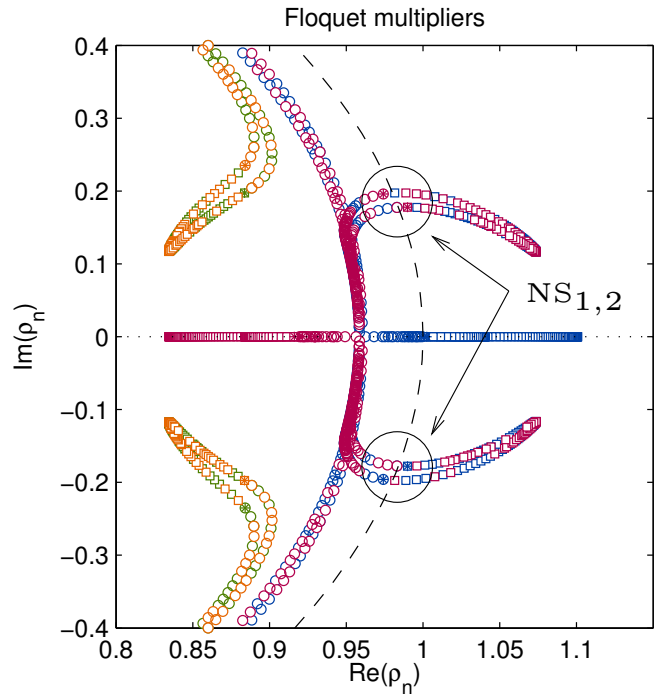
$$\begin{cases} u_1(t) \simeq a_1 \cos(\Omega t + \varphi_1) \\ u_2(t) \simeq a_2 \cos(2\Omega t + \varphi_2) \end{cases}$$



Strong coupling with the 2nd. harm. of mode 2

Excitation of the first mode: bifurcations

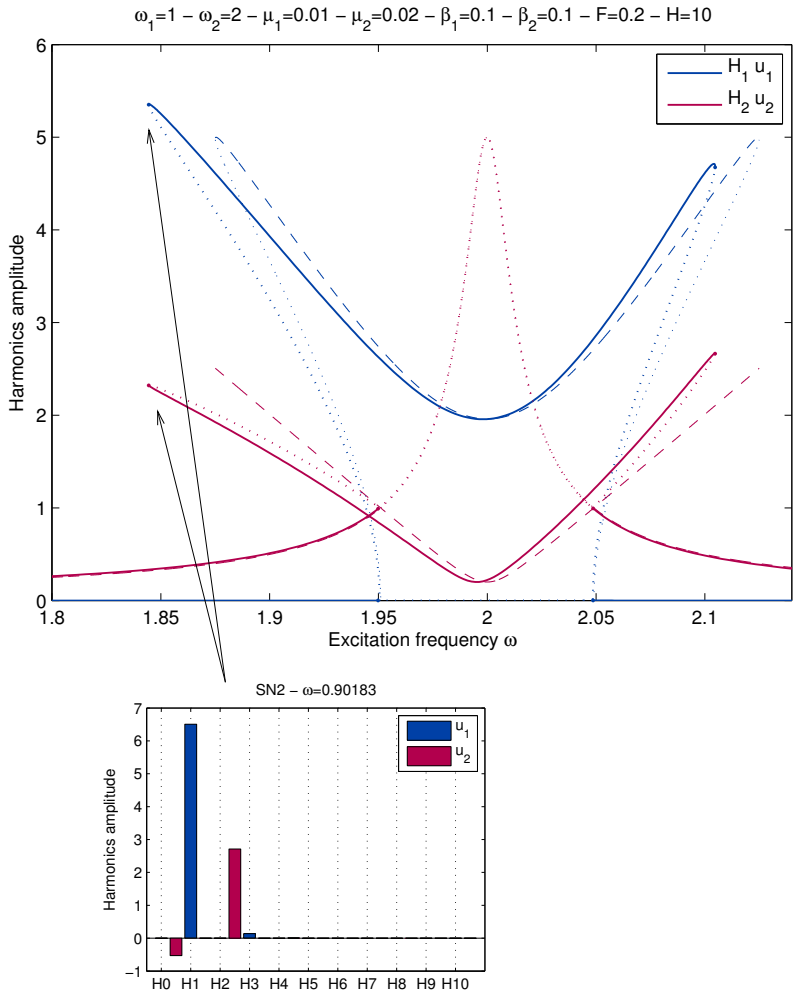


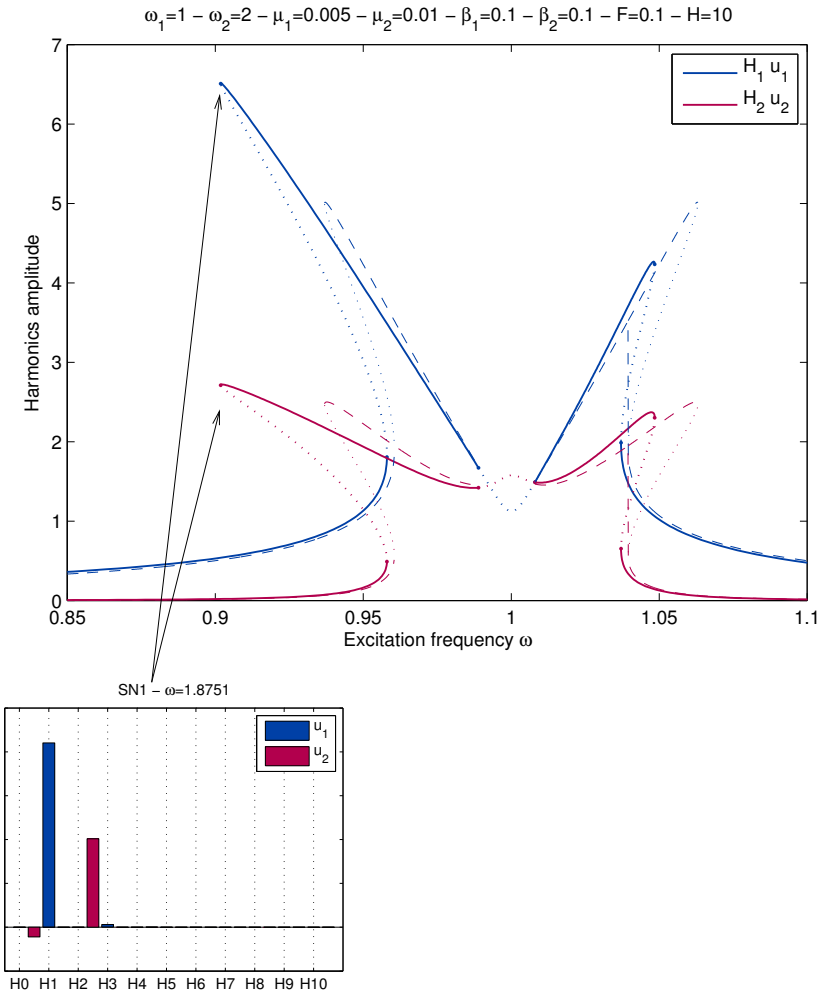


▷ **Comparisons with:** the 1st. order MS. solution + AUTO (not shown)

The two Neimark-Sacker bifurcations are well predicted

Comparisons with the M.S. 1st. order solution





- ▷ **Fully validated with AUTO**
- ▷ **Numerical solution more precise than multiple scale solution**