Practice the quadratic recasts of Dynamical systems for *Manlab-4*

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Quadratic recasts of Differential Equations

Example 1 : basic, simplified forced pendulum

Main equation (ξ, M, k, F are fixed) :

$$\ddot{\theta} + \xi\dot{\theta} + (\theta - \frac{\theta^3}{6}) + k(\theta - \frac{\pi}{M}) = F\cos(\lambda t)$$

Definition of the auxiliary variables :

$\psi =$

Quadratic recast of the main equations :

$R_1 :=$

Quadratic equations defining the auxiliary variables :

$R_{aux1} :=$

Total vector of unknowns and vector of equations :

$U_f = [\theta \; \psi \; \lambda]^T$

$R_f = [R_1 \; R_{aux1}]^T$

$\text{Forcing}(t) = [\qquad\qquad\qquad\qquad]^T$

Example 2 : basic, simplified forced pendulum

Main equation (ξ, M, k, ω are fixed) :

$$\ddot{\theta} + \xi\dot{\theta} + (\theta - \frac{\theta^3}{6}) + k(\theta - \frac{\pi}{M}) = \lambda \cos(\omega t)$$

Definition of the auxiliary variables :

$$\psi =$$

Quadratic recast of the main equations :

$$R_1 :=$$

Quadratic equations defining the auxiliary variables :

$$R_{aux1} :=$$

Total vector of unknowns and vector of equations :

$$U_f = [\theta \ \psi \ \lambda]^T$$

$$R_f = [R_1 \ R_{aux1}]^T$$

$$\text{Forcing}(t) = [\qquad\qquad\qquad\qquad\qquad\qquad]^T$$

Example 3 : basic, pendulum

Main equations (k,M are fixed) :

$$r(\ddot{\theta}, \dot{\theta}, \theta, \lambda) := \ddot{\theta} + \sin(\theta) + k(\theta - \frac{\pi}{M}) = 0$$

Add the unfolding term :

$$r(\ddot{\theta}, \dot{\theta}, \theta, \lambda) := \ddot{\theta} + \lambda\dot{\theta} + \sin(\theta) + k(\theta - \frac{\pi}{M}) = 0$$

Definition of the auxiliary variables :

Quadratic recast of the main equations :

$$R_1 :=$$

Equations defining the auxiliary variables, with the (quadratic) differential form for the transcendental definitions :

$R_{aux1} :=$ $\qquad$ $dR_{aux1} :=$

$R_{aux2} :=$ $\qquad$ $dR_{aux2} :=$

Total vector of unknowns and vector of equations :

$$U_f = [\qquad]^T$$

$$R_f = [R_1 \; R_{aux1} \; R_{aux2}]^T$$

$$dR_f = [0 \; dR_{aux1} \; dR_{aux2}]^T$$

$$\text{Forcing}(t) = [0 \; 0 \; 0]^T$$

Example 4 : your favorite

Main equations :

Definition of the auxiliary variables :

Recast of the main equations (with isolated forcing term) :

Equations defining the auxiliary variables, with the (quadratic) differential form for the transcendental definitions :

Total vector of unknowns and vector of equations with the forcing isolated :

$U_f = [\qquad\qquad\qquad\qquad\qquad\qquad]^T$

$R_f = [\qquad\qquad\qquad\qquad\qquad\qquad]^T$

$dR_{aux} = [\qquad\qquad\qquad\qquad\qquad]^T$

$\text{Forcing}(t) = [\qquad\qquad\qquad\qquad\qquad\qquad]^T$

ODE example for Equilibrium, Periodic Solutions and stability

Example 5 : Van der Pol

Second order differential equation of a Van der Pol oscillator :

$$\ddot{x} + \xi\dot{x} - \lambda(1 - x^2)\dot{x} + x = 0$$

Main equations written on the form $r(z) := f(z) - \dot{z} = 0$:

Definition of the auxiliary variables :

$$z_{a1} =$$

$$z_{a2} =$$

Quadratic recast of the main equations :

$$R_1 =$$

$$R_2 =$$

Quadratic equations defining the auxiliary variables :

$$R_{aux1} =$$

$$R_{aux2} =$$

Total vector of unknowns and vector of equations :

$$U_f = [\qquad\qquad\qquad\qquad \lambda]^T$$

$$R_f = [R_1\ R_2\ R_{aux1}\ R_{aux2}]^T$$

$$\text{Forcing}(t) = [0\ 0\ 0\ 0]^T$$

Example 6 : Forced pendulum

Second order differential equation of a forced pendulum :

$$\ddot{\theta} + \xi\dot{\theta} + \sin(\theta) = F\cos(\lambda t)$$

Main equations written on the form $r(z,t) := f(z) - \dot{z} = \text{Forcing}(t)$:

Definition of the auxiliary variables :

Quadratic recast of the main equations :

Equations defining the auxiliary variables, with the (quadratic) differential form for the transcendental definitions :

Total vector of unknowns and vector of equations with the forcing isolated :

$$U_f = [\qquad\qquad\qquad\qquad\qquad\qquad]^T$$

$$R_f = [\qquad\qquad\qquad\qquad\qquad\qquad]^T$$

$$dR_{aux} = [\qquad\qquad\qquad\qquad\qquad]^T$$

$$\text{Forcing}(t) = [\qquad\qquad\qquad\qquad\qquad\qquad]^T$$