Practice the quadratic recasts of Algebraic systems for *Manlab-4*

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Quadratic recasts of algebraic systems

Example 1 : basic, without function

Main equations (M is fixed) :

$$r(\theta, \lambda) := \theta \; - \; \frac{\theta^3}{6} \; + \; \lambda(\theta - \frac{\pi}{M}) = \; 0$$

Definition of the auxiliary variables :

$$\psi =$$

Quadratic recast of the main equations :

$$R_1 :=$$

Quadratic equations defining the auxiliary variables :

$$R_{aux1} :=$$

Total vector of unknowns and vector of equations :

$$U_f = [\theta \; \lambda \; \psi]^T$$

$$R_f = [R_1 \; R_{aux1}]^T$$

Example 2 : simple, with fraction

Main equations (k is fixed) :

$$r(\theta, \lambda) := \theta \; - \; \frac{\theta^3}{6} \; + \; k(\theta - \frac{\pi}{\lambda}) = \; 0$$

Definition of the auxiliary variables :

Quadratic recast of the main equations :

$$R_1 :=$$

Quadratic equations defining the auxiliary variables :

Total vector of unknowns and vector of equations :

$$U_f =$$

$$R_f =$$

Example 3 : basic, with function

Main equations (k is fixed) :

$$r(\theta,\lambda) := \sin(\theta) + k(\theta - \frac{\pi}{\lambda}) = \ 0$$

Definition of the auxiliary variables :

Quadratic recast of the main equations :

$$R_1 :=$$

Equations defining the auxiliary variables, with the (quadratic) differential form for the transcendental definitions :

$$R_{aux1} := \qquad dR_{aux1} :=$$

$$R_{aux2} := \qquad dR_{aux2} :=$$

$$R_{aux3} :=$$

Total vector of unknowns and vector of equations :

$$U_f =$$

$$R_f = [R_1 \ R_{aux1} \ R_{aux2} \ R_{aux3}]^T$$

$$dR_f = [0 \ dR_{aux1} \ dR_{aux2} \ 0]^T$$

Example 4 : to summarize (without functions)

Main equations (μ is fixed) :

$$r_1(u_1, u_2, \lambda) := 2u_1 - u_2 + 100\,\frac{u_1}{1+u_1+u_1^2} - \lambda = 0$$
$$r_2(u_1, u_2, \lambda) := 2u_2 - u_1 + 100\frac{u_2}{1+u_2+u_2^2} - \lambda - \mu = 0$$

Definition of the auxiliary variables :

Quadratic recast of the main equations :

$$R_1 :=$$

$$R_2 :=$$

Quadratic equations defining the auxiliary variables :

Total vector of unknowns and vector of equations :

Example 5 : to summarize (with functions)

Main equations :

$$
\begin{aligned}
r_1(u_1, u_2, \lambda) &= u_1 + \lambda \tfrac{\exp(u_2)}{1+u_1} \\
r_2(u_1, u_2, \lambda) &= u_2 + u_1 \tanh(\tfrac{-5u_1}{1+u_1u_2})
\end{aligned}
$$

Definition of the auxiliary variables :

Quadratic recast of the main equations :

Equations defining the auxiliary variables, with the (quadratic) differential form for the transcendental definitions :

Total vector of unknowns and vector of equations, with differentiated forms if needed :

Exemple 6 : your favorite system

Main equations :

Definition of the auxiliary variables :

Quadratic recast of the main equations :

Equations defining the auxiliary variables, with the (quadratic) differential form for the transcendental definitions :

Total vector of unknowns and vector of equations, with differentiated forms if needed :

$$U_f = [\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad]^T$$

$$R_f = [\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad]^T$$

$$dR_f = [\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad]^T$$

Example 7 : Euler schemes, vectorial implementation

Main equations (h is a constant) for $0 \leq n \leq N \in \mathbb{N}$ and for $0 \leq \lambda \leq 1$ we write $\theta_{n+\lambda} = (1-\lambda)\theta_n + \lambda\theta_{n+1}$. The integration of the pendulum-spring can be written :

$$\begin{array}{llll} \theta_0 & = \frac{\pi}{4} & \text{and} & \theta_{n+1} & = \theta_n + h\dot{\theta}_{n+\lambda} \\ \dot{\theta}_0 & = 0 & \text{and} & \dot{\theta}_{n+1} & = \dot{\theta}_n + h\left(-\sin(\theta_{n+\lambda}) - k(\theta_{n+\lambda} - \frac{\pi}{M})\right) \end{array}$$

Definition of the auxiliary variables :

Quadratic recast of the main equations :

Equations defining the auxiliary variables, with the (quadratic) differential form for the transcendental definitions :

Total vector of unknowns and vector of equations, with differentiated forms if needed :

$$U_f = [\qquad\qquad\qquad\qquad]^T$$

$$R_f = [\qquad\qquad\qquad\qquad]^T$$

$$dR_f = [\qquad\qquad\qquad\qquad]^T$$